Physical and Mathematical Foundations of Quantum Mechanics

PHY 307 - Fall 2016

Stony Brook University Department of Physics and Astronomy, College of Arts and Sciences

Course Instructor: **Alfred Scharff Goldhaber** goldhab@max2.physics.sunysb.edu, 631 632 7975, fax 631 632 7954. Office Math 6-113. (Twitter)

Lectures: Tu-Th 2:30-3:50 p.m., Frey 305

Recitations: Sec. 1 M 12:00 p.m.- 12:53 p.m., Physics P-123. Sec. 2 M 2:30 p.m.- 3:23 p.m., Physics P-123. Course credits: Four credits corresponding to four contact hours. Office Hours: To be determined, but principally by appointment at class, on the phone, or on email.

Teaching Assistant and Grader: To be determined.

INTRODUCTION:

This fall will be the 9th offering of PHY 307, a course which to the best of my knowledge is unique in the world. It is based on my idea of a physics that might have been, but didn't happen in fact. Albert Einstein's first paper in his "miracle year" 1905 was the only one that he himself called 'revolutionary': Commonly called 'the photo-electric effect' paper (or more simply the 'photo-effect paper'), its actual title was 'Concerning an heuristic point of view toward the emission and transformation of light'. It was indeed revolutionary, because by that time a century of physics theory and experiment had seemed to establish beyond any doubt that light is a wave phenomenon, and not, as Newton had suggested, made of particles. Einstein did not know how to reconcile his proposal that light is made of localizable clumps that he called 'quanta' with the wave picture, but he was able to use his idea to explain and also to predict many phenomena of the absorption of light by atoms or molecules in materials leading to the emission of electrons. For almost 20 years, Einstein was the only important physicist to believe his claim, which we now know is essential to the description of electromagnetic phenomena at microscopic scales. As a result, his work had no direct influence on the development of quantum theory until de Broglie in 1924 proposed a wave theory for electrons, which until then had been universally described as particles. It only took one more year for first Heisenberg and then Schrödinger to provide superficially different but mathematically equivalent descriptions of modern quantum mechanics, which have survived unscathed since that time.

The aim of this course, like Einstein's paper, is to provide a heuristic path that short-cuts two decades of development, by showing how Einstein's particle idea has to be right, and how it can be reconciled with the Maxwell wave theory of light. This can be beneficial for students in two ways. First, it allows a coherent and natural way to quantum mechanics. Secondly, in the course of the development students encounter many of the physical and mathematical concepts they will need in their further work on the subject. This is in contrast to the traditional "modern physics" approach, that does not directly motivate quantum mechanics (even if it is introduced without direct motivation), followed by what is essentially an 'axiomatic' course, with again no explicit motivation beyond the fact that 'it works'. Thus my hope for the course, that many of my past students have felt succeeded, has been that it provide a natural path to the quantum mechanics of a single photon. From there, everything else can be built up in a logical way.

Interestingly, the root of this demonstration lies in Einstein's other, quickly accepted, work of 1905, the special theory of relativity. By that time, the idea of absolute conservation of total energy for an isolated system was well accepted as part of the bedrock of physics. In principle, one might imagine that energy could disappear in one place and simultaneously appear somewhere far away. However, in a different velocity frame the energy would first disappear and only later reappear, or vice versa. This would mean total energy is NOT conserved. Thus the validity of special relativity implies that energy must be locally conserved, meaning that whatever disappears from one box must flow into immediately neighboring boxes. Planck had produced a hybrid picture of the quantum mechanics of light plus matter. The oscillators or atoms inside matter had a quantum relation between energy and frequency, but the only reason such a relation also applied to light was that the radiation from matter had to be emitted in quantum multiples of $h\nu$ – it still would be in classical electromagnetic waves, whose energy density is given by expressions in terms of electric and magnetic fields derived in classical Maxwell theory.

This is the point where Einstein could have used the notion of local energy conservation to conclude that, for very dilute light (on the scale of wavelength), the classical energy density expression could not possibly be correct. An atom absorbs energy from the light and becomes excited in a very short time. If the energy were described by the classical expression, then that energy would have to be absorbed into the atom from a region very large compared to the speed of light multiplied by the transition time. This would be grossly

NON-local conservation of energy, thus not acceptable. Conclusion: the classical electromagnetic energy density must be wrong in this limit, and the actual energy must be concentrated in geometrically small packets. That leaves open the question of how, if at all, the classical energy density is relevant. The answer we now know is that this classical energy density divided by hν gives the number density of quanta or photons. It was that principle, which Einstein clearly felt intuitively, that allowed him to make so many successful predictions about the photo-electric excitation of atoms in materials to release electrons.

One can only wonder how history might have been different if Einstein had made this reasoning explicit. As it was, when he was awarded the Nobel prize in 1922 (for the year 1921), still no one else believed his quanta hypothesis, though the successes of his predictions were undeniable. One may wonder if Einstein's omission of reasoning based on local conservation of energy should be called his greatest failure.

INTENDED AUDIENCE:

As described below, this course is designed to be fully accessible to students who have completed one year of introductory physics and the associated introductory calculus [If Modern Physics, PHY 251, is taken at the same time, the two courses should reinforce each other; the same applies for PHY 300, PHY 301, and PHY 405]. For such students the intent is that they should feel as if they were "gliding into quantum mechanics." As with hang gliding, first it is necessary to climb to a high place. In this case the "high place" is the properties of electromagnetic waves, well worth mastering in their own right. For capable sophomores who do well in PHY 307 the way is open to take PHY 308, Quantum Physics, in Spring 2014, even if they lack pre-requisites for 308, because in the past students in this position also have done well in 308. This would be a one-year 'speedup' in entering quantum physics, putting our program on a par with many other college physics programs.

Students who already have more background, including Waves and Optics (PHY 300) and Electromagnetic Theory I, II (PHY 301, 302) as well as Quantum Physics (PHY 308) and Advanced Quantum Physics (PHY 405) can benefit as well, because important and subtle concepts they have seen already are approached from a different perspective, reinforcing the learning. At the end, we'll consider what might or might not have been the practical effect if Albert Einstein already had constructed one-photon quantum mechanics in 1905.

COURSE DESCRIPTION:

Maxwell waves and their properties: Intensity, energy density, momentum density. Complementary descriptions of wave super-positions in position and in "wave number." Planck-Einstein relation between energy and frequency for light quanta. Einstein-De Broglie relation between momentum and wavelength. Number density and probability density of photons. One-photon quantum mechanics, **with Maxwell field as the wave function**. Diffraction phenomena. Uncertainty relation between wave number and position, hence between momentum and position. Schrödinger Equation. Three lecture hours and one recitation hour per week.

Course pre- and co-requisites

Pre-requisites -- Introductory classical mechanics and electromagnetism. Introductory differential and integral calculus. {Advisory co-requisite -- Multivariable calculus. Because the Maxwell equations involve three space dimensions, and time, for the vector electric and magnetic fields, they give a wonderful laboratory for working with multivariable calculus. Having that course beforehand or concurrently would give extra and possibly complementary perspective on these properties, but all that is needed will be worked out fully in PHY 307.} Course numbers: PHY 121-2, or 125-7, or 131-2, or 141-2, MAT 131-2 or 141-2 or 125-7 or 171 or AMS 151-161. {Advisory co-requisite: MAT 203, MAT 205, AMS 261, or equivalent.} Students who took and passed Physical and Mathematical Foundations of Quantum Mechanics under the previous course numbers PHY 390 and PHY 274 are ineligible to take PHY 307. PHY 307, while not required for the major like other courses including PHY 251 and PHY 308, is intended to provide a "keystone" which stabilizes and advances the entire arc of undergraduate quantum studies.

MOTIVATION:

For more than eight decades the teaching of quantum mechanics has followed two main tracks. The first approach is roughly historical. There is some logic to that, because if people originally found things in a certain sequence then it is at least possible for students to learn in that sequence. This is the style of "modern physics" courses, which give a broad survey of phenomena especially in microscopic physics, including many of the steps in the somewhat contorted path that led to modern quantum mechanics.

The other method is axiomatic, exemplified by Dirac's great book on the subject, a pattern followed by many more recent texts. In my opinion there is a missing piece in this traditional two-tiered approach. The intellectual jump from the broad survey to the axiomatic treatment is not easily articulated, and can be disconcerting for students as they try to negotiate the transition. The approach of the present course could be called "quasi-historical," meaning to build on a history that might have been if Einstein in 1905 had shown even greater audacity than he did in his "photoelectric effect" paper. The aim is to provide a path that is as short and direct as possible from 1905 to 1925-6, when Heisenberg and Schrödinger introduced the theory we still have today.

If short, this climb can be quite challenging, and requires an intense study of Maxwell's electromagnetic waves and their properties to provide the foundation for the ascent. The take-home message, as in Einstein's special theory of relativity, is that when one tries to fit together Maxwell's theory with other knowledge, in case of doubt one should defer to Maxwell. This may seem natural when one recognizes that Maxwell's is the first fundamental field theory in physics, still unaltered at the classical level nearly 150 years after its formulation. Thus the new course complements not only the existing one on modern physics but also other departmental offerings on aspects of electromagnetism and light. The path, as indicated in the syllabus below, goes through Maxwell electrodynamics to one-photon quantum mechanics, and uses that as a base to develop one-electron quantum mechanics.

I believe this approach could help overcome what seems to me a deficiency in current physics training, that students are exposed to classical mechanics and classical electrodynamics twice during their undergraduate years, but a systematic approach to quantum physics comes along only at the end of junior and beginning of senior year. My idea is that this should be a first-term sophomore course, simultaneous with and complementing the Modern Physics course, allowing students to enter PHY 308, Quantum Physics, in the spring of sophomore year. In every previous year PHY 307 has been an example of "inter-age learning" – the students are a mixture of sophomores, juniors, and seniors, with each cohort adding something valuable to the experience. As mentioned already, for sophomores going into PHY 308 in the spring it would be a one-year speed-up compared to the current pattern here, and would put our program on the same pace (with respect to quantum physics) as many other colleges. Colleagues at a number of other institutions have introduced quantum mechanics through light rather than through massive particles like electrons, but I'm unaware of any textbook that does this. The goal of PHY 307 then, is after learning about electromagnetic waves and wave packets to recognize these solutions as quantum-mechanical waves describing the behavior of photons, or particles of light.

EXPECTED OUTCOMES:

Quantum Mechanics often is called counterintuitive, or even 'weird'. The main take-home message from this course should be that all the apparent weirdness stems from the wave aspects, which are less familiar to most of us than particle behaviors. Here are some important examples, with which students should gain experience during the course:

1. Superposition. When two classical waves overlap in some region, the total wave amplitude is simply the sum of the amplitudes of the two waves. This property carries over to the wave functions studied in quantum mechanics.

2. The energy density in a classical wave is proportional to the square of the wave amplitude. In quantum mechanics the same holds for the probability density for finding a particle— it is proportional to the square of the corresponding wave function.

3. Unlike observable classical waves, such as water waves or sound waves or light waves, quantum wave functions are intrinsically complex, and therefore cannot be observed directly.

4. Classical waves and their quantum counterparts obey definite equations determining how they change in time. In other words, they are predictable.

5. Classical and quantum waves both can show polarization. That is, in addition to their direction of motion, they can carry intrinsic directional information, which for the quantum case can be identified with intrinsic angular momentum, or 'spin'.

6. The more sharply pinned down in spatial location be a classical wave, the broader the range of 'wave number' associated with it. In quantum mechanics this becomes the Uncertainty Principle, forcing a tradeoff between measurement accuracies at a given time for the position and the momentum of a particle.

7. Entanglement: Two classical waves can be correlated, for example, when they arise from scattering of a single wave from a half-silvered mirror. As a result, information about the one wave gives constraints on where the other wave is more or less intense. This property again carries over into quantum mechanics.

COURSE REQUIREMENTS:

Attendance and Homework Policy One-quarter point per class for attendance after the add-drop period, and one-sixth point for recitation attendance. [Recitations begin in the first week of class.] Half-credit for late arrival. Up to five points for each homework assignment. Late homework will be graded down 20% for each day missed. [That means after 5 days there will be no credit, but the paper may still be graded to give feedback to the student.] Both for homework and for attendance excused lateness or absence will not count against the student. Homework normally will be assigned on a Tuesday (posted on Blackboard) and due in class Tuesday two weeks later (if there is no regular class on a Tuesday, the homework will be due on the Thursday following).

Required and recommended books Two required textbooks (#1 and #2), and one recommended book (#3) from the Schaum Outline Series:

1. Electromagnetics, 4th Edition, Joseph A. Edminster, McGraw Hill 2013.
2. Advanced Mathematics for Engineers and Scientists, 2nd Edition, Murray R. Spiegel, McGraw Hill 2009.
3. Quantum Mechanics, 2nd edition, Yoav Peleg, Reuven Pnini, Elyahu Zaarur and Eugene Hecht, Mc Graw Hill 2010.

There will be no assignments from these books, but they contain much useful information about topics discussed in the course. If we come to a topic not discussed in one of the books, I shall provide additional references and/or background material, besides what is offered in lecture.

Exams None.

Paper and Presentation Each student is asked to prepare a 5-page paper and make a 5-minute class presentation about the topic of the paper. If two people work together on a paper and presentation, each will get the same grade, but then the paper and presentation should be twice as long, for three people three times as long, etc. The topic of the paper and presentation should be a biography of someone who appears in the historical background for quantum mechanics [In other words, NOT one of the people from Planck on who directly made quantum physics]. A list of possible subjects will be distributed early in the course. All papers should both describe the life and the work of the scientist who did the work. Students may choose a subject from the list (first-come first served), or propose a different subject for approval. The stages in the process will be submitted in hard copy in class as indicated:

1. Submission of subject choice: Thursday 8 September.
2. One or two paragraph abstract AND detailed (meaning at least 7 items) outline: Thursday 22 September.
3. Notes filling out each item in the outline, and possibly adding more: Thursday 20 October. The notes should have everything you would need (if you were alone on a desert island) to write a complete final draft. That means you should have all your references [A minimum of three references should be to published work, whether books or articles in refereed journals. Besides these, references to websites also may be included.], and any pictures you want to show. The idea is that there are two big pushes in creating your paper. The first is gathering all the thoughts and information that you need, and the second is to put this into a coherent, carefully written essay:
4. Final draft -- This should be a polished paper, created from the informal but complete notes. The final draft should be submitted not only in hard copy in class, but also electronically to SafeAssign: Thursday 10 November.
5. Presentations -- these will be given during regular lecture class hours beginning Tuesday 24 November. There will be guidelines to assure that everything can be done during the available time.

Journal Entries Very shortly after each lecture, you are asked to go over your notes, then put them aside, and write a paragraph or two in the appropriate day's Journal on Blackboard, describing what you learned including anything you felt needed more explanation. This has two benefits. For students, there is evidence that doing such an exercise fixes the information in your mind more effectively than, for example, studying hard just before an exam. That's especially good here because we don't have exams! For the teacher, it gives a good chance to find out what is getting across and what is not. Please try to write coherent sentences. <u>Lists of formulas don't really show that you understand something</u>, so formulas only should appear as illustrations of what is in your text. Journal entries should be completed by 12 midnight of the night following each class. The “20% rule” also applies here for each day’s delay.

Engagement in recitations (and the course generally) During recitation, a student or group of students who raise an interesting question will get one half-point credit each. One kind of question that is welcome though it will NOT get credit is "Could you go over a particular problem (or some item in lecture)?" To get the credit, you need to describe something you have done. It might be, for example, that you tried to do a problem in a particular way and ran into some difficulty, and you want help in understanding that difficulty. Equally well, it might be that you have a proposed solution, and are presenting it. In lecture, it might be saying that my lecture statements seem to imply such and such, which doesn’t make sense, or that they seem to imply so and so, which is an interesting extension of my presentation. Raising a question about a discussion in one of the reference texts also is a good way to earn engagement credit (for this, it would be most helpful to email me in advance of a recitation session). I emphasize engagement for the recitations because they should be very much "live time" that involves active learning.

Grading Ten homework sets during the term, with a possible 5 points for each set. This adds up to a possible 50 points for the semester. One-quarter point for attendance at each lecture, and one-sixth point for attendance at each recitation, after the end of the add-drop period (i.e., beginning Wednesday 9 September). This adds up to a possible 8 points for the semester. Lateness cuts the grade for a particular attendance in half. Excused absence or lateness will not be penalized. Up to 20 points for the paper and presentation. One half point extra credit for each student on each homework set for which two to four people have cooperated in discussing the homework, and then written up the solutions separately (up to 5 points for the semester). To receive this credit, EACH OF the students cooperating must list the names of OTHER members of the group on the TOP RIGHT FRONT of the homework paper, taking care to make clear who is the student submitting that particular paper by writing the SUBMITTER'S name on the TOP LEFT FRONT. Up to 28 points for Journal entries after each of the 28 lecture sessions, including the student presentations. Up to 4 points for demonstrating engagement, by asking detailed questions about (or presenting solutions of) homework or raising interesting questions related to lecture or textbook subjects. This may be done in recitation, by email, in journal entries, or in lecture. Excluding extra credit, this adds up to 110 points for the semester. Letter grades: >105=A, >100 =A-, >95=B+, >90=B, >85=B-, >80=C+, >75=C, >70=C-. Actual letter grades will not be lower than implied by these guidelines, but might be higher.

MEETING SCHEDULE

[NOTE: FROM PREVIOUS EXPERIENCE, SOME OF THESE ITEMS MAY TAKE MORE THAN ONE LECTURE -- THERE IS A RESERVE OF AT LEAST TWO LECTURE PERIODS TO ACCOMMODATE THIS]

Lecture plan:

1. Perspective on the course, procedures
2. Particle dynamics and conservation laws
3. Wave dynamics
4. Complex functions, Gaussian function, Gaussian integral, Fourier transforms, dual

descriptions of waves
5. Wave packets
6. Pythagorean theorem and vectors in spaces of arbitrary dimension
7. Gauss law for charge and electric field, Ampère law for steady current and magnetic field
8. Faraday induction law for magnetic and electric fields
9. Local conservation of electric charge
10. Maxwell addition to Ampère law
11. Differential form of Maxwell equations, wave solutions
12. Polarization of light, "magical" behavior of polarizing elements, 2X2 matrices
13. Lorentz force law and local conservation of energy and momentum
14. Electromagnetic intensity, energy density, momentum density
15. Photon energy (Planck-Einstein relation), intensity, number density, momentum
16. Probability density for a one-photon system
17. Diffraction patterns
18. Relation of energy and momentum to time and space derivatives
19. Derivation of Schrödinger equation for non-relativistic particle
Role of potential energy function, and contrast with refraction of light
20. What would have happened if Einstein had developed this approach in 1905?
21. Correlations of indistinguishable particles -- Bose-Einstein and Fermi-Dirac statistics
22. Dirac equation, hole theory, connection of spin and statistics, CPT theorem
23. Entanglement

Recitations: The first meetings of the recitations will take place the FIRST WEEK, and will be for getting acquainted and exploring student hopes for the course. Already at this stage it should be possible to earn credit for engagement. After that, in a week with homework due the following day, questions and discussions about the homework will be the principal business, though other questions about lecture material, including at the end of the term presentations by other students, also will be welcome. As mentioned above, it's expected that students, individually or in groups, will present at least 4 questions (or relevant comments) during the term, giving the possibility of earning up to 4 points for engagement with the material. Students may volunteer in advance to present an issue with a problem, or to present a solution to a problem, or to raise a question relating to lecture or textbooks. If you want to check on the validity of your point, you may discuss it with me by email in advance of the class. I may ask students to share with the class issues raised in their journal entries, and this also could produce engagement credit. Besides preparation for solving the homework, which works best if students have tried it before recitation, these sessions give a chance to go beyond what has been discussed in lecture. If students have ideas about how to make the recitations more rewarding, I'm happy to listen (and to recognize productive suggestions with engagement credit!).

CLASS PROTOCOL

Cell phone and electronic device statement: Everyone, including the instructor, is permitted to bring devices to class, but they must be in quiet mode. Under normal circumstances incoming calls should not be answered during class. Students are free to make audio or video recordings of the class, and are permitted to post these recordings on the blackboard site. Our lecture classroom is equipped for video-recording, and the video-record will be made available through Blackboard. In the past, students have said that these recordings are useful not only for a missed class but also for going over difficult points in lectures.

CLASS RESOURCES

Library resources: Some texts will be put on reserve in the Math-Physics library. Blackboard will be used extensively, and students are strongly encouraged to post comments or questions relating to the class, anonymously if they wish. Of course, to get engagement credit for an apt comment or question, you do need to say who you are!

DISABILITY SUPPORT SERVICES (DSS) STATEMENT

If you have a physical, psychological, medical or learning disability that may impact your course work, please contact Disability Support Services, ECC (Educational Communications Center) Building, room128, (631) 632-6748. They will determine with you what accommodations, if any, are necessary and appropriate. All information and documentation is confidential. See http://studentaffairs.stonybrook.edu/dss/index.shtml

Students who require assistance during emergency evacuation are encouraged to discuss their needs with their professors and Disability Support Services. For procedures and information go to the following website: http://www.stonybrook.edu/ehs/fire/disabilities

ACADEMIC INTEGRITY STATEMENT

Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty are required to report any suspected instances of academic dishonesty to the Academic Judiciary. For more comprehensive information on academic integrity, including categories of academic dishonesty, please refer to the academic judiciary website at http://www.stonybrook.edu/uaa/academicjudiciary/

For this course there are some extra aspects associated with two complementary approaches, cooperation and independence. If you cooperate in discussing the homework, you should indicate with whom you worked, but you should be independent in writing up the solutions. That means it is quite possible that two students who discussed a particular problem could get different grades for their solutions. On the papers and presentations it is crucial to indicate all sources for significant statements you make. If you are quoting somebody, you should put the statement in quotation marks and indicate where you got it. If your source is a web presentation, you should indicate where it is (i.e., the URL), and the author if any is given, but you also should check it, as there are many incorrect statements in web presentations. Something completely cooperative is group papers and presentations. Something completely individual is your journal entry, indicating your sense of the content of each lecture.

CRITICAL INCIDENT MANAGEMENT

Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of Judicial Affairs any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, or inhibits students' ability to learn.